

Formelsammlung Maschinendynamik

Version: 1.1

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Dieses Dokument wurde mit L^AT_EX gesetzt.

1 Kinematik

1.1 Laufgrad

2 Grundlagen der Schwingungstechnik

2.1 Erzwungene Schwingung bei harmonischer Anregung

Kraftanregung der Masse:

$$\ddot{y}(t) + 2D \cdot \dot{y}(t) + w_0^2 \cdot y(t) = \frac{F(t)}{m} = \frac{F_0(t)}{m} \cdot \cos\Omega t$$

Fußpunktanregung über Feder:

$$\ddot{y}(t) + 2D \cdot \dot{y}(t) + w_0^2 \cdot y(t) = w_0^2 \cdot u_0 \cdot \cos\Omega t$$

Fußpunktanregung über Dämpfer:

$$\ddot{y}(t) + 2D \cdot \dot{y}(t) + w_0^2 \cdot y(t) = -2D \cdot w_0 \cdot \Omega \cdot u_0 \cdot \sin\Omega t$$

Fliehkraftanregung:

$$\begin{aligned} \ddot{y}(t) + 2D \cdot \dot{y}(t) + w_0^2 \cdot y(t) &= \frac{m_u \cdot e}{m} \cdot w_0 \cdot \Omega^2 \cdot \sin\Omega t \\ &= -\frac{m_u \cdot e}{m} \cdot w_0 \cdot \Omega^2 \cdot j \cdot e^{j\Omega t} \end{aligned}$$

Fußpunktanregung über Feder und Dämpfer:

$$\ddot{y}(t) + 2D \cdot \dot{y}(t) + w_0^2 \cdot y(t) = 2D \cdot w_0 \cdot \dot{u}(t) + w_0^2 \cdot u(t) = 2D \cdot w_0 \cdot \dot{u}(t) + w_0^2 \cdot u_0 \cdot \cos\Omega t$$

Anregung durch Beschleunigungskräfte:

$$m \cdot \ddot{y}(t) = -r \cdot [\dot{y}(t) - \dot{u}(t)] - c \cdot [y(t) - u(t)]$$

2.2 Auswertung

Eigenkreisfrequenz:	$w_0 = \sqrt{\frac{c}{m}}$
Dämpfung:	$D = \frac{r}{2 \cdot w_0 \cdot m}$
Abklingkonstante:	$\delta = D \cdot w_0$
Eigenkreisfrequenz gedämpftes System:	$w_D = w_0 \cdot \sqrt{1 - d^2}$
Abstimmung:	$\eta = \frac{\Omega}{\omega}$
Amplituden-Frequenzgang 1:	$\alpha_1(\eta) = \frac{1}{\sqrt{(1 - \eta^2)^2 + 4D \cdot \eta^2}}$
Phasenverschiebung 1:	$\phi_{p1} = \arctan\left(\frac{2D\eta}{1 - \eta^2}\right)$
Amplituden-Frequenzgang 2:	$\alpha_2(\eta) = \frac{2D\eta}{\sqrt{(1 - \eta^2)^2 + 4D \cdot \eta^2}} = 2D\eta \cdot \alpha_1(\eta)$
Phasenverschiebung 2:	$\phi_{p2} = -\arctan\left(\frac{1 - \eta^2}{2D\eta}\right)$
Amplituden-Frequenzgang 3:	$\alpha_3(\eta) = \frac{\eta^2}{\sqrt{(1 - \eta^2)^2 + 4D \cdot \eta^2}} = \eta^2 \cdot \alpha_1(\eta)$
Phasenverschiebung 3:	$\phi_{p3} = \arctan\left(\frac{2D\eta}{1 - \eta^2}\right) = \phi_{p1}$
Amplituden-Frequenzgang 4:	$\alpha_4(\eta) = \frac{\sqrt{1 + 4D^2\eta^2}}{\sqrt{(1 - \eta^2)^2 + 4D \cdot \eta^2}} = \sqrt{1 + 4D^2\eta^2} \cdot \alpha_1(\eta)$
Phasenverschiebung 4:	$\phi_{p4} = -\arctan\left(\frac{2D\eta^3}{1 - \eta^2 + 4D^2\eta^2}\right)$
Max. der Amplitude:	$y_{pmax} = \frac{F_0}{c} \cdot \alpha_1(\eta)$

3 Lineare Schwingungssysteme

3.1 Ungedämpfte Eigenschwingung

Schwingungsverhalten beschrieben durch homogene DGL:

$$\begin{aligned} M \cdot \ddot{x}(t) + S \cdot x(t) &= 0 \\ m_{ij} \cdot \ddot{x}(t) + s_{ij} \cdot x(t) &= 0 \end{aligned}$$

Eigenschwingansatz:

$$\begin{aligned} x &= \hat{x} \cdot e^{j\omega t} \\ x_j &= \hat{x}_j \cdot e^{j\omega t} \end{aligned}$$

3.2 Gedämpfte Eigenschwingung 1

$$\begin{aligned} M \cdot \ddot{x}(t) + D \cdot \dot{x}(t) + S \cdot x(t) &= 0 \\ m_{ij} \cdot \ddot{x}(t) + d_{ij} \cdot \dot{x}(t) + s_{ij} \cdot x(t) &= 0 \end{aligned}$$

Reeller Ansatz:

$$x_j = \hat{x}_j \cdot e^{\lambda \cdot t}$$

3.3 Gedämpfte Eigenschwingung 2

$$\begin{aligned} \ddot{x} + \frac{k}{m} \cdot \dot{x} + \frac{c}{m} \cdot x &= 0 \\ \ddot{x} + 2D\omega_0 \cdot \dot{x} + \omega_0^2 \cdot x &= 0 \\ \ddot{x} + 2\delta \cdot \dot{x} + \omega_0^2 \cdot x &= 0 \end{aligned}$$

3.4 Charakteristische Gleichung

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$|\lambda \cdot M - S| = 0 \quad |\omega^2 \cdot M - S| = 0$$

$$\lambda = \omega^2 \text{ (Eigenwert)}$$

3.5 Definitionen

Lehssches Dämpfungsmass / Dämpfungsgrad: $D = \frac{\delta}{\omega_0} = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

$$D = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}}$$

Abklingkonstante:

$$\delta = D \cdot \omega_0 = \sqrt{\omega_0^2 - \omega_d^2}$$

Logarithmisches Dämpfungsinkrement:

$$\Lambda = \delta \cdot T_d = \frac{1}{n} \left(\frac{x(t)}{x(t + nT_d)} \right) = \frac{2\pi D}{\sqrt{1 - D^2}}$$

Abstimmung:

$$\eta = \frac{\omega_d}{\omega_0} = \sqrt{1 - 2D^2}$$

Schwingungsdauer: $T = \frac{2\pi}{\omega}$

Winkelgeschwindigkeit: $\omega = 2\pi n$

Steifigkeit von Wellen: $F = C \cdot \varphi$ mit $C = \frac{G \cdot I_t}{l}$ (Vollwelle: $I_t = \frac{\pi}{32} d^4$)

Balken: $f = \frac{F \cdot l^3}{48 \cdot EI} \Rightarrow F_C = C \cdot f = \frac{48 \cdot EI}{l^3} \cdot y$

4 Auswahl der Verfahren

- reell und symmetrisch
- reell und unsymmetrisch
- komplex, hermitesch, d.h. konjugiert transponiert zur komplexen Matrix
- komplex und nicht hermitisch

4.1 Jacobi-Verfahren

Beruhet auf Hauptsachsentheorem, es besagt, dass zu jeder symmetrischen Matrix **A** eine orthogonale Transformationsmatrix **T** existiert

4.2 Simultane Vektoraddition

Für symmetrische und positiv definierte Matrizen

5 Verschiedenes

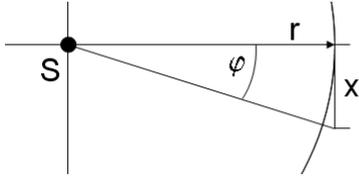
5.1 Aufstellen BGL:

$$\text{Kraft} = m \cdot \ddot{x}$$

$$\text{Feder} = c \cdot x$$

$$\text{Daempfer} = d \cdot \dot{x}$$

$$\text{Traegheitsmoment} = J \cdot \ddot{\varphi}$$



$$x = r \cdot \varphi$$

$$\varphi = \frac{x}{r}$$

Massenträgheitsmomente ausgewählter Körper

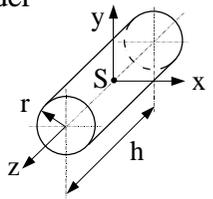
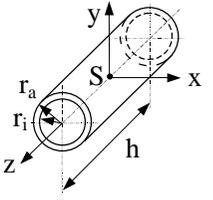
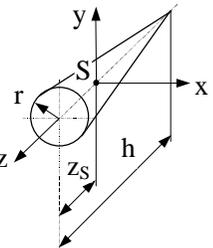
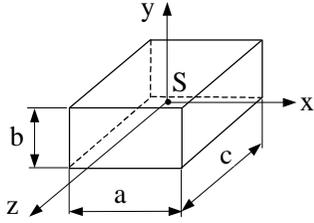
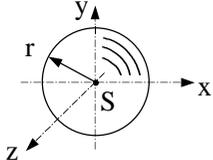
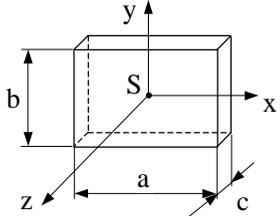
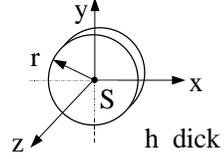
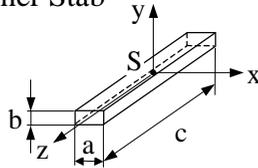
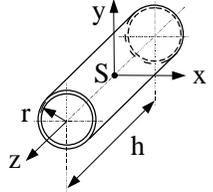
<p>Zylinder</p> 	$m = \rho p h r^2$ $J_z = \frac{1}{2} m r^2$ $J_x = J_y = \frac{1}{4} m \left(r^2 + \frac{1}{3} h^2 \right)$
<p>Hohlzylinder</p> 	$m = \rho p h (r_a^2 - r_i^2)$ $J_z = \frac{1}{2} m (r_a^2 + r_i^2)$ $J_x = J_y = \frac{1}{4} m \left(r_a^2 + r_i^2 + \frac{1}{3} h^2 \right)$
<p>Kreiskegel</p> 	$z_S = \frac{1}{4} h$ $m = \frac{1}{3} \rho p h r^2$ $J_z = \frac{3}{10} m r^2$ $J_x = J_y = \frac{3}{80} m (4r^2 + h^2)$
<p>Quader</p> 	$m = \rho a b c$ $J_z = \frac{1}{12} m (a^2 + b^2)$ $J_y = \frac{1}{12} m (a^2 + c^2)$ $J_x = \frac{1}{12} m (b^2 + c^2)$
<p>Kugel</p> 	$m = \frac{4}{3} \rho p r^3$ $J_x = J_y = J_z = \frac{2}{5} m r^2$
<p>dünne Rechteckscheibe</p> 	$c \ll a, b$ $J_z = \frac{1}{12} m (a^2 + b^2)$ $J_y \approx \frac{1}{12} m a^2$ $J_x \approx \frac{1}{12} m b^2$
<p>dünne Kreisscheibe</p> 	$h \ll r$ $J_z = \frac{1}{2} m r^2$ $J_x = J_y \approx \frac{1}{4} m r^2$
<p>dünner Stab</p> 	$a, b \ll c$ $J_z \approx 0$ $J_x = J_y \approx \frac{1}{12} m c^2$
<p>dünnwandiger Hohlzylinder</p> 	$r_a \approx r_i \approx r$ $J_z \approx m r^2$ $J_x = J_y \approx \frac{1}{2} m \left(r^2 + \frac{h^2}{6} \right)$

Abbildung 1: Massenträgheitsmomente ausgewählter Körper

Flächenmomente 2. Ordnung (Auswahl)

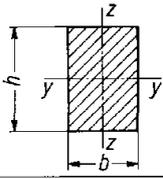
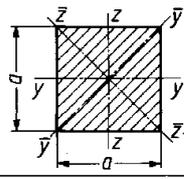
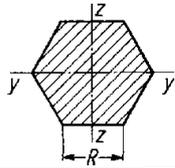
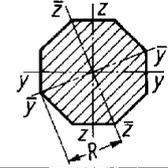
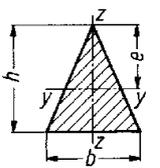
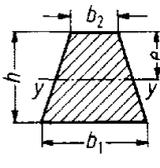
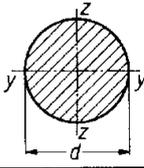
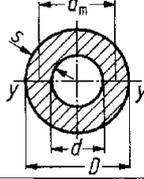
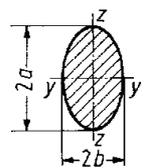
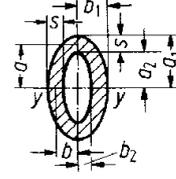
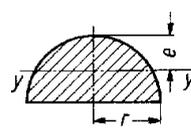
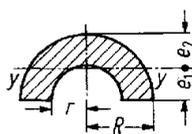
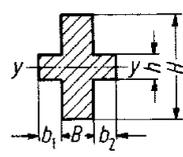
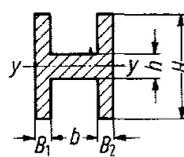
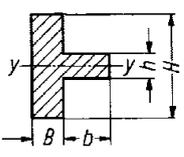
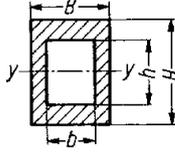
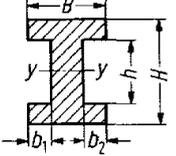
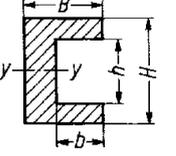
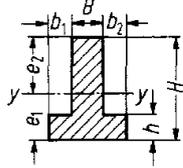
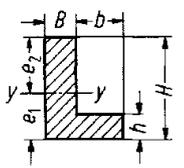
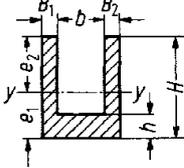
	$I_y = \frac{bh^3}{12}$ $I_z = \frac{hb^3}{12}$ $W_y = \frac{bh^2}{6}$ $W_z = \frac{hb^2}{6}$		$I_y = I_z = \frac{a^4}{12}$ $W_y = W_z = \frac{a^3}{6}$ $I_y = I_z = \frac{a^4}{12}$ $W_y = W_z = \frac{\sqrt{2}}{12} a^3 = 0,118 a^3$
	$I_y = I_z = \frac{5\sqrt{3}}{16} R^4 = 0,5413 R^4$ $W_y = \frac{5}{8} R^3 = 0,625 R^3$ $W_z = \frac{5\sqrt{3}}{16} R^3 = 0,5413 R^3$		$I_y = I_z = (1+2\sqrt{2}) \frac{R^4}{6} = 0,638 R^4$ $W_y = W_z = 0,6906 R^3$ $I_y = I_z = (1+2\sqrt{2}) \frac{R^4}{6} = 0,638 R^4$ $W_y = W_z = 0,638 R^3$
	$I_y = \frac{bh^3}{36}$ $I_z = \frac{hb^3}{48}$ $W_y = \frac{bh^2}{24} \text{ für } e = \frac{2}{3} h$ $W_z = \frac{hb^2}{24}$		$I_y = \frac{h^3}{36} \frac{b_1^2 + 4b_1b_2 + b_2^2}{b_1 + b_2}$ $W_y = \frac{h^2}{12} \frac{b_1^2 + 4b_1b_2 + b_2^2}{2b_1 + b_2}$ <p>für $e = \frac{h}{3} \frac{2b_1 + b_2}{b_1 + b_2}$</p>
	$I_y = I_z = \frac{\pi d^4}{64}$ $W_y = W_z = \frac{\pi d^3}{32}$		$I_y = I_z = \frac{\pi (D^4 - d^4)}{64}$ $W_y = W_z = \frac{\pi (D^4 - d^4)}{32D}$ <p>bei geringer Wanddicke $\left(\frac{s}{d_m}\right)^2 \ll 1$:</p> $I_y = I_z = \frac{\pi d_m^3 s}{8}, W_y = W_z = \frac{\pi d_m^2 s}{4}$
	$I_y = \frac{\pi a^3 b}{4}$ $I_z = \frac{\pi b^3 a}{4}$ $W_y = \frac{\pi a^2 b}{4}$ $W_z = \frac{\pi b^2 a}{4}$		$I_y = \frac{\pi}{4} (a_1^3 b_1 - a_2^3 b_2)$ $W_y = \frac{\pi (a_1^2 b_1 - a_2^2 b_2)}{4 a_1}$ <p>bei geringer Wanddicke:</p> $I_y = \frac{\pi a^2 (a+3b)s}{4}, W_y = \frac{\pi a (a+3b)s}{4}$
	$I_y = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = 0,1098 r^4$ $W_y = I_y / e = 0,1908 r^3$ <p>für $e = \left(1 - \frac{4}{3\pi}\right) r = 0,5756 r$</p>		$I_y = 0,1098(R^4 - r^4) - 0,283R^2 r^2 \frac{R-r}{R+r}$ $W_{y1,2} = I_y / e_{1,2}$ <p>für $e_1 = \frac{4}{3\pi} \frac{R^2 + Rr + r^2}{R+r}$ bzw. $e_2 = R - e_1$</p>
			$I_y = \frac{BH^3 + bh^3}{12}$ $W_y = \frac{BH^3 + bh^3}{6H}$ <p>mit $B = B_1 + B_2$ $b = b_1 + b_2$</p>
			$I_y = \frac{BH^3 - bh^3}{12}$ $W_y = \frac{BH^3 - bh^3}{6H}$ <p>mit $b = b_1 + b_2$</p>
			$I_y = \frac{BH^3 + bh^3}{3} - (BH + bh) e_1^2$ <p>mit $B = B_1 + B_2, b = b_1 + b_2$ $W_{y1,2} = I_y / e_{1,2}$ für $e_1 = \frac{1}{2} \frac{BH^2 + bh^2}{BH + bh}$ bzw. $e_2 = H - e_1$</p>

Abbildung 2: Flächenmomente 2. Ordnung

Integrationstabelle - Kraftgrößenverfahren

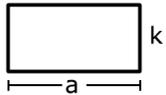
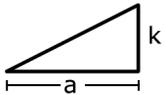
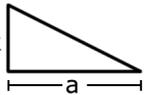
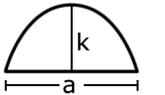
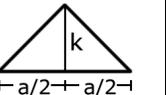
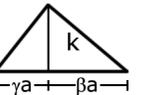
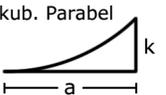
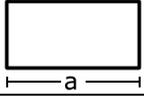
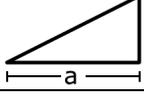
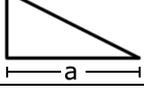
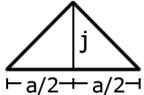
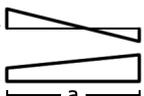
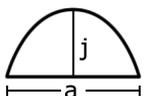
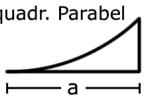
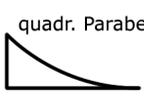
		a	b	c	d	e	f	g	h
	$\int \frac{\overline{MM}}{EI} ds$								
1		ajk	$\frac{1}{2}ajk$	$\frac{1}{2}ajk$	$\frac{2}{3}ajk$	$\frac{1}{2}ajk$	$\frac{1}{2}ajk$	$\frac{1}{2}aj(k_1 + k_2)$	$\frac{1}{4}ajk$
2		$\frac{1}{2}ajk$	$\frac{1}{3}ajk$	$\frac{1}{6}ajk$	$\frac{1}{3}ajk$	$\frac{1}{4}ajk$	$\frac{1}{6}ajk(1 + \gamma)$	$\frac{1}{6}aj(k_1 + 2k_2)$	$\frac{1}{5}ajk$
3		$\frac{1}{2}ajk$	$\frac{1}{6}ajk$	$\frac{1}{3}ajk$	$\frac{1}{3}ajk$	$\frac{1}{4}ajk$	$\frac{1}{6}ajk(1 + \delta)$	$\frac{1}{6}aj(2k_1 + k_2)$	$\frac{1}{20}ajk$
4		$\frac{1}{2}ajk$	$\frac{1}{4}ajk$	$\frac{1}{4}ajk$	$\frac{5}{12}ajk$	$\frac{1}{3}ajk$	$\frac{1}{12}ajk \frac{3 - 4\gamma^2}{\delta}$ <i>für $\gamma \leq \delta$</i>	$\frac{1}{4}aj(k_1 + k_2)$	$\frac{3}{32}ajk$
5		$\frac{1}{2}a(j_1 + j_2)k$	$\frac{1}{6}a(j_1 + 2j_2)k$	$\frac{1}{6}a(2j_1 + j_2)k$	$\frac{1}{3}a(j_1 + j_2)k$	$\frac{1}{4}a(j_1 + j_2)k$	$\frac{1}{6}a[j_1(1 + \delta) + j_2(1 + \gamma)]k$	$\frac{1}{6}a[j_1(2k_1 + k_2) + j_2(k_1 + 2k_2)]$	$\frac{1}{20}ak(j_1 + 4j_2)$
6		$\frac{2}{3}ajk$	$\frac{1}{3}ajk$	$\frac{1}{3}ajk$	$\frac{8}{15}ajk$	$\frac{5}{12}ajk$	$\frac{1}{3}ajk(1 + \gamma\delta)$	$\frac{1}{3}aj(k_1 + k_2)$	$\frac{2}{15}ajk$
7		$\frac{1}{3}ajk$	$\frac{1}{4}ajk$	$\frac{1}{12}ajk$	$\frac{1}{5}ajk$	$\frac{7}{48}ajk$	$\frac{1}{12}ajk(1 + \gamma + \gamma^2)$	$\frac{1}{12}aj(k_1 + 3k_2)$	$\frac{1}{60}ajk$
8		$\frac{1}{3}ajk$	$\frac{1}{12}ajk$	$\frac{1}{4}ajk$	$\frac{1}{5}ajk$	$\frac{7}{48}ajk$	$\frac{1}{12}ajk(1 + \delta + \delta^2)$	$\frac{1}{12}aj(3k_1 + k_2)$	

Abbildung 3: Integrationstabelle - Kraftgrößenverfahren